

# APPLICATION AND DESIGN GUIDELINES OF THE PML ABSORBER FOR FINITE ELEMENT SIMULATIONS OF MICROWAVE PACKAGES

J. Gong\*, S. Legault\*, Y. Botros\*, J.L. Volakis\* and P. Petre†

\* Radiation Laboratory

Dept. of Elect. Engin. & Comp. Sci.

University of Michigan, Ann Arbor, MI 48109-2122

† Compact Software Inc.

201 McLean Blvd, Paterson, N.J. 07504

## Abstract

The recently introduced perfectly matched layer(PML) uniaxial absorber for frequency domain finite element simulations has several advantages. In this paper we present the application of PML for microwave circuit simulations along with design guidelines to obtain a desired level of absorption. Different feeding techniques are also investigated for improved accuracy.

## Introduction

In the numerical simulation of 3D microwave circuits using partial differential approaches, it is necessary to terminate the domain with some type of non-reflective boundary conditions. When using frequency domain PDE formulations, such as the finite element method, the standard approach is to employ some type of absorbing boundary conditions(ABCs) [1], [2], [3]. Also, the use of infinite elements [4] or port conditions [5] have been investigated. All of these mesh truncation methods require *a priori* knowledge of the dominant mode fields

and, to a great extent, their success depends on the purity of the assumed mode expansion at the mesh truncation surface. Larger computational domains must therefore be used and the accuracy of the technique in computing the scattering parameters could be compromised.

Recently, a new anisotropic (uniaxial) absorber [6] was introduced for truncating finite element meshes. This absorber is reflectionless(i.e. perfectly matched at its interface) for all incident waves, regardless of their incidence angle and propagation constants. As a result, it can be placed very close to the circuit discontinuity and is particularly attractive for terminating the computational domain of high density microwave circuits where complex field distributions could be present.

Although the proposed uniaxial PML absorber has a perfectly matched interface, in practice a finite metal-backed (say) layer must be used which is no longer reflectionless due to the presence of the pec (see Fig. 1). It is therefore of interest to optimize the absorptivity of the layer by proper selection of the parameters to achieve a given reflectivity with a minimum layer thickness. In this paper, we present

TU  
3C

guidelines for implementing the PML absorber to truncate finite element meshes in microwave circuit simulations. Example microwave circuit calculations are also given to demonstrate the accuracy of the PML absorber and the FEM simulator. More examples will be presented at the conference.

## Absorber Design

An extensive study was carried out using two-dimensional (see Fig. 1) and three dimensional models (see Fig. 2) in order to optimize the absorber's performance using the minimum thickness and discretization rate. As expected, the absorber's thickness, material properties and the discretization rate all play an equally important role on the performance of the PML. The typical field behavior interior to the absorber is shown in Fig. 3. As seen, for small  $\beta$  values the field decay is not sufficient to eliminate reflections from the metal backing. For large  $\beta$  values, the rapid decay can no longer be accurately modeled by the FEM simulation and consequently the associated VSWR increases to unacceptable values. However, an optimum value of  $\beta$  which minimizes the reflection coefficient for a given layer thickness and discretization can be found. The parameters  $\beta$  and  $t$  play complementary roles and the study shows that the PML absorber's performance can be characterized in terms of the product  $\frac{\beta t}{\lambda_g}$  (a scalable quantity when  $\alpha = 0$ ) and the discretization rate. A two-dimensional analysis was carried out to determine the optimum values of  $\frac{\beta t}{\lambda_g}$  and  $N$  (the number of samples in the PML layer) for maximum absorption near normal incidence. It was determined that given a desired reflection coefficient  $|R|$  for the PML absorber, the optimum  $\frac{\beta t}{\lambda_g}$  and  $N$  values are approximately given by the

expressions [7]

$$\begin{aligned}\frac{\beta t}{\lambda_g} &= -0.0106|R| + 0.0433 \\ N &= 0.147 \exp \left[ 7.353 \frac{\beta t}{\lambda_g} \right]\end{aligned}$$

where  $|R|$  must be given in dB and  $N$  is equal to or exceeding the right hand value. As an example, if we desire to have a value of  $|R|$  equal to  $-50\text{dB}$ , from the above formulae we have that  $\frac{\beta t}{\lambda_g} \approx 0.58$  and  $N = 10$ . It should be noted that though the design formulae were derived with  $\alpha = 0$  they also hold for small non-zero values of  $\alpha$ .

## 3D Modeling Examples

The PML performance as predicted by the formulae was investigated by using it to truncate the domain of 3-D microwave circuits. For example, Fig. 4 shows the optimum value of  $\frac{\beta t}{\lambda_g} \approx 0.96$  obtained from the above design equations compares well with the results of the full wave FEM analysis of the microstrip line shown in Fig. 2. The 3-D FEM computations were carried out using  $N = 5$  for modeling the PML absorber across its thickness and from the given formulae, it follows that  $R = -41\text{dB}$  and this agrees well with the optimum value shown in Fig. 4. Another example is the meander line shown in Fig. 5. For the FEM simulation, the structure was placed in a rectangular cavity of size  $5.8\text{mm} \times 18.0\text{mm} \times 3.175\text{mm}$ . The cavity was tessellated using  $29 \times 150 \times 5$  edges and only 150 edges were used along the y-axis. The domain was terminated with a 10 layer PML, each layer being of thickness  $t = 0.12\text{mm}$ . The  $S_{11}$  results are shown in Fig 6 and are in good agreement with the measured data [8].

## References

- [1] R.L. Higdon, "Absorbing boundary conditions for acoustic and elastic waves in stratified media," *J. Comp. Phys.*, Vol. 101, pp. 386-418, 1992.
- [2] T.B.A. Senior and J.L. Volakis, *Approximate Boundary Conditions in Electromagnetics*, IEE Press, London, 1995.
- [3] J-S. Wang and R. Mittra, "Finite Element Analysis of MMIC structures and electronic packages using absorbing boundary conditions," *IEEE Trans. Microwave Th. and Techn.*, Vol. 42, pp. 441-449, March 1994.
- [4] S. Tsitsos, A. Gibson and A. McCormick, "Higher Order Modes in Coupled Striplines: Prediction and Measurement," *IEEE Trans. Microwave Th. and Techn.*, Vol. 42, pp. 2071-2077, Nov. 1994.
- [5] J.F. Lee, "Analysis of Passive Microwave Devices by Using three-dimensional tangential vector finite elements," *Int. J. Num. Model.: Electronic Net., Dev. and Fields*, Vol. 3, pp. 235-246, 1990.
- [6] Z.S. Sacks, D.M. Kingsland, R. Lee and J.F. Lee, "A perfectly matched anisotropic absorber for use as an absorbing boundary condition," to appear in *IEEE Trans. Antennas Propagat.*
- [7] S. Legault, T.B.A. Senior and J.L. Volakis, "Design of Planar Absorbing Layers for Domain Truncation in FEM Applications," submitted to *Electromagnetics*.
- [8] I. Wolf, "Finite Difference Time-domain Simulation of Electromagnetic Fields and Microwave Circuits," *International Journal of Microwave and Millimeter-Wave Computer-Aided Engineering*, August 1992.

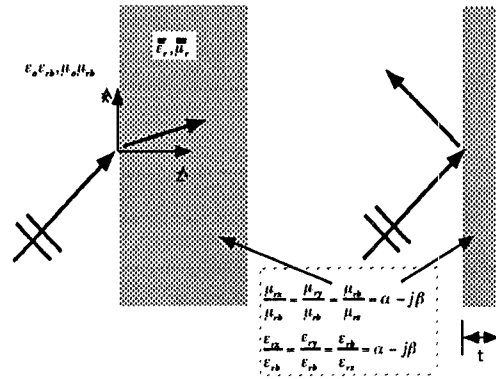


Figure 1: Illustration of wave incidence upon a perfectly match interface (PML) with and without metal backing.

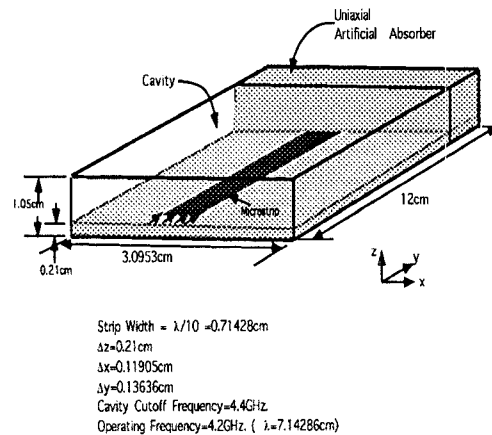


Figure 2: Shielded microstrip line terminated by a perfectly matched uniaxial absorber layer.

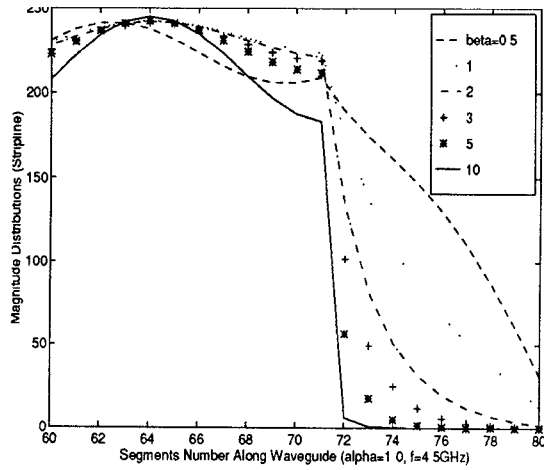


Figure 3: Illustration of the field decay pattern inside the PML layer.

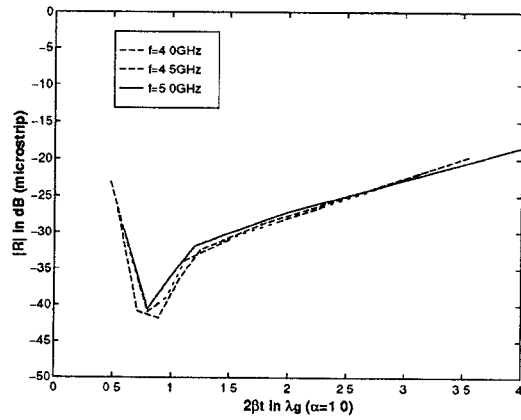


Figure 4: Reflection coefficient vs  $2\beta t/\lambda_g$  with  $\alpha=1$ , for the shielded microstrip line terminated by the perfectly matched uniaxial layer.

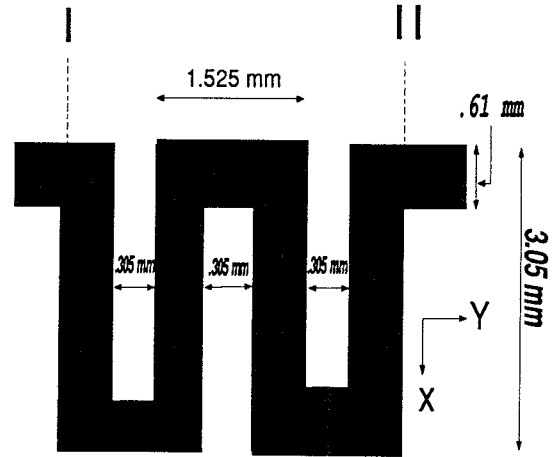


Figure 5: Illustration of a meander line geometry used for comparison with measurement.

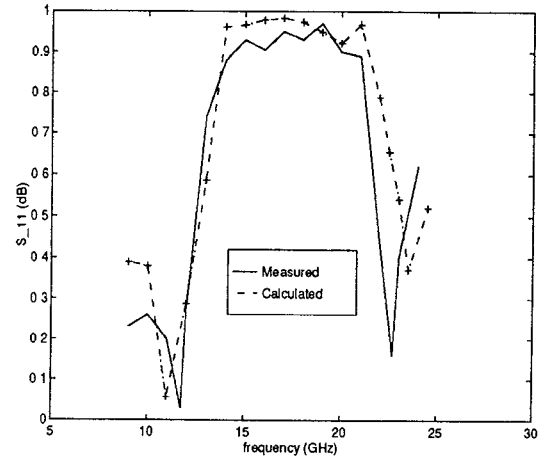


Figure 6: Comparison of calculated and measured results for the meander line shown in Fig.5.